2.2 M-ary Baseband Pulse Amplitude Modulation (PAM)

M-ary baseband pulse amplitude modulation is a one-dimensional signal set with basis function

\[ \phi_0(t) = p(t) \]  \hspace{1cm} (2.20)

where \( p(t) \) is any unit energy pulse such as those described in Appendix B. The notation developed in the preceding sections is extended to denote a symbol sequence with a new symbol occurring every \( T_s \) seconds. The resulting PAM signal is

\[ s(t) = \sum_n a(n)p(t - nT_s) \]  \hspace{1cm} (2.21)

where the subscript “0” has been dropped on the symbol value \( a \) and an index \( n \) has been added to denote the \( n \)-th symbol in a sequence of symbols. The subscript \( n \) on the summation symbol means that the sum is over \( n \) and may be finite (for a finite data sequence) or infinite if an infinite data sequence is assumed. When it is important, limits on the summation will be explicit.

Some samples constellations for baseband PAM are shown in Figure 2.6. Note that except for \( M = 2 \), the constellations points do not all have the same energy. For \( M = 4 \) the average symbol energy is

\[ E_{avg} = \frac{(-3)^2 + (-1)^2 + (+1)^2 + (+3)^2}{4} = 5 \]  \hspace{1cm} (2.22)

and for \( M = 8 \) the average symbol energy is

\[ E_{avg} = \frac{(-7)^2 + (-5)^2 + (-3)^2 + (-1)^2 + (+1)^2 + (+3)^2 + (+5)^2 + (+7)^2}{8} = 21. \]  \hspace{1cm} (2.23)

For the general case of M-ary PAM, the constellation points are evenly spaced along the constellation axis at locations

\[ -M + 1, -M + 3, \ldots, -1, +1, \ldots, M - 3, M - 1 \]  \hspace{1cm} (2.24)

The average energy of the general M-ary PAM constellation is

\[ E_{avg} = \frac{M^2 - 1}{3}. \]  \hspace{1cm} (2.25)

For each constellation, the minimum Euclidean distance is 2.
Figure 2.6: Baseband PAM constellations for $M = 2$ (a), $M = 4$ (b), $M = 8$ (c), and general $M$, (d).
2.2.1 Continuous-Time Realization

The PAM modulator using continuous-time processing is illustrated in the upper portion of Figure 2.7. The input is a serial bit stream where new bits arrive every $T_b$ seconds. Bits are converted to $\log_2 M$ bit symbols by the serial to parallel converter which outputs a new symbol every $T_s = \log_2(M) \times T_b$ seconds. Thus, the symbol rate is $1/(\log_2 M)$ times the bit rate. The $\log_2 M$ bits form the address to a single look-up table that stores the symbol values specified by the constellation. Conceptually, the series of look-up table outputs forms a weighted impulse train

$$i(t) = \sum_n a(n) \delta(t - nT_s) \tag{2.26}$$

that drives the pulse shaping filter with impulse response $p(t)$. The output of the pulse shaping filter is the pulse train given by (2.21).

The matched filter PAM detector is illustrated in the lower portion of Figure 2.7. The received waveform is the impulse train (2.21) plus noise. The matched filter is a filter whose impulse response is a time reversed version of the pulse shape as described in Section 2.1.4. The matched filter output, $x_0(t)$ is sampled at $T_s$-spaced intervals to produce a sequence of signal space projections that are used for detection. Detection uses the decision rule (2.14). The detector presented in Figure 2.7 is an idealized system. It is assumed that the time instant the matched filter output is to be sampled is known precisely. In a real system, this knowledge is provided by a symbol timing synchronization subsystem.

The mathematical expressions for these operations are as follows. Let

$$r(t) = s(t) + w(t) \tag{2.27}$$

be the received signal where $s(t)$ is the PAM pulse train given by (2.21) and $w(t)$ is the additive noise\(^1\). The matched filter output is

$$x_0(t) = r(t) * p(-t) = \int_{T_1 + t}^{T_2 + t} r(\lambda)p(\lambda - t)d\lambda. \tag{2.28}$$

Using (2.27) and (2.21), the matched filter output is

$$x_0(t) = \sum_l a(l) \int_{T_1 + t}^{T_2 + t} p(\lambda - lT_s)p(\lambda - t)d\lambda + \int_{T_1 + t}^{T_2 + t} w(\lambda)p(\lambda - t)d\lambda \tag{2.29}$$

\(^1\)The noise term $w(t)$ will be modeled as a zero-mean white Gaussian random process with power spectral density $N_0/2$ Watts/Hz.
which may be expressed as

\[ x_0(t) = \sum_l a(l)r_p(lT_s - t) + v(t) \]  \hspace{1cm} (2.30)

where \( v(t) \) is the output of the matched filter due to the additive noise at the input (the second integral in (2.29)) and \( r_p(\tau) \) is the pulse shape autocorrelation function defined by

\[ r_p(\tau) = \int_{T_1}^{T_2} p(t)p(t - \tau)dt \]  \hspace{1cm} (2.31)

as described in Appendix B.

The matched filter output is sampled at \( t = kT_s \) to produce the signal space projection

\[ x_0(kT_s) = \sum_l a(l)r_p((l - k)T_s) + v(kT_s). \]  \hspace{1cm} (2.32)

As discussed in Appendix B, the autocorrelation function for all full response unit-energy pulse shapes satisfies

\[ r_p(mT_s) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}. \]  \hspace{1cm} (2.33)

Condition (2.33) also holds for partial response pulse shapes that satisfy the Nyquist no-ISI condition. As a consequence (2.32) may be expressed as

\[ x_0(kT_s) = a(k) + v(kT_s). \]  \hspace{1cm} (2.34)

This shows that the signal space projection consists of the true point plus a noise term. The effect of the noise term is to perturb the position of \( x_0(kT) \) in the signal space. The symbol decision follows the rule (2.14) or the equivalent form given by (??) as described in Section ???. The detector produces an erroneous output (i.e. a symbol error) when the noise term forces \( x_0(kT_s) \) to be closer to a point in the signal space other than the point corresponding to the waveform that was sent. The probability of error for M-ary PAM is derived in Section 2.2.3.

An important conceptual tool is a modulo-\( T_s \) plot of the matched filter output \( x_0(t) \). Examples of these plots for the NRZ, MAN, HS, and SRRC pulse shapes are shown in Figure 2.8. Note that the time axis on the plots are shifted so that the optimum sampling instant occurs in the middle of the plot. The overlay is used to present all the possible trajectories of \( r_p(\tau) \) as determined by the symbol sequence. These plots for the HS and SRRC pulse shapes resemble a human eye. for this reason, these plots are called eye diagrams.
2.2 M-ary Baseband Pulse Amplitude Modulation (PAM)

Figure 2.7: A PAM modulator (top) and detector (bottom) using continuous-time processing.
Figure 2.8: Eye diagrams for (a) the NRZ pulse shape, (b) the MAN pulse shape, (c) the HS pulse shape, and (d) the SRRC pulse shape with a 0.5 roll-off factor.
2.2.2 Discrete-Time Realization

The PAM modulator using discrete-time processing is illustrated in the upper portion of Figure 2.9. The input bit stream is segmented into non-overlapping \( \log_2 M \) bit symbols which access \( M \)-ary symbol values in the same manner as the PAM modulator based on continuous-time processing. Here the pulse shaping is performed in discrete-time by using an FIR filter whose impulse response consists of \( T \)-spaced samples of the pulse shape \( p(t) \). The symbol sequence \( \{ \ldots, a(n-2), a(n-1), a(n), \ldots \} \) is converted to a discrete-time impulse train by upsampling by \( N \) where \( N \) is the ratio of sample rate to symbol rate:

\[
N = \frac{T_s}{T}. \quad (2.35)
\]

The upsampling operation inserts \( N - 1 \) zeros in between each symbol as shown. Interpolation is performed by the pulse shaping filter to produce a discrete-time pulse train as shown. The discrete-time pulse train is converted to a continuous-time pulse train by the ADC for transmission over the waveform channel.

Three clock rates are used in the modulator. The first clock rate is the bit rate clock used to clock the bits into the serial-to-parallel converter. The output of the serial-to-parallel converter is clocked at \( 1/\log_2(M) \) times the bit rate. This is the symbol rate clock. The pulse shaping occurs at a rate that is \( N \) times the symbol clock rate. At a minimum the high sample rate should satisfy the sampling theorem. In practice, this rate is as high as the system constraints will allow it to be.

The PAM detector using discrete-time processing is illustrated in the lower portion of Figure 2.9. The received signal is the transmitted pulse train (2.21) plus noise. \( T \)-spaced samples of \( r(t) \) are produced by the ADC. These samples are filtered by a matched filter whose impulse response consists of \( T \)-spaced samples of the time reversed pulse shape. The resulting matched filter output is given by

\[
x_0(nT) = \sum_{m=\frac{T_2}{T}+n}^{\frac{T_2}{T}} r(mT)p((m-n)T)
\]

where \( T_1 \) and \( T_2 \) define the time support of the pulse shape \( p(t) \). The matched filter output is downsampled at \( n = k \frac{T_s}{T} \) to produce the sequence of signal space projections \( x_0(kT_s) \). The \( k \)-th symbol estimate is made using the decision rule (2.14).

This detector is an idealized system in two ways. First, it is assumed that the ADC produces an integer number of samples/symbol. In other words, that \( T_s/T \) is an integer. Since this assumption is rarely true in practice, a resampling filter is often included in the system block diagram. The second assumption is the exact symbol timing instant is known and coincides with a sample. In reality, this will not be the case and a discrete-time symbol synchronizer must be used to estimate the phase and frequency of the downsampling operation.
There are at least two clock rates in this system. The first clock rate is the sampling rate used by the ADC to sample the received waveform. The sample rate must satisfy the sampling theorem. Unlike the modulator, however, there is little need to sample the signal any higher than the minimum rate given by the sampling theorem. The matched filtering occurs at the high clock rate and the output of the matched filter is downsampled by \( N \) to one sample/symbol. The clock rate here is the symbol rate. Symbol decisions are made at the clock rate. There is a third clock rate, the bit clock, if the symbol decisions are mapped to \( \log_2 M \) bits.

The mathematical expressions for the receiver operations mimic those for the continuous-time case. In this text, signal energies and correlations are referenced to continuous-time since they were defined for the continuous-time signals. For example, the pulse correlation function can be expressed in terms of \( T \)-spaced samples of \( p(t) \) using the backward difference approximation for integration:

\[
\rho_p(\tau) = \int_{T_1}^{T_2} p(t)p(t - \tau)dt \\
\approx T \sum_{n=T_1/T}^{T_2/T} p(nT)p(nT - \tau).
\]  

From this the relationship between the signal energy in continuous-time and discrete-time is

\[
E_m = \int_{T_1}^{T_2} a_m^2 p^2(t)dt \\
= a_m^2 \rho_p(0) \\
\approx a_m^2 T \sum_{n=T_1/T}^{T_2/T} p^2(nT).
\]

The \( T \)-spaced samples of the received signal are

\[
r(nT) = s(nT) + w(nT) = \sum_{l} a(l)p(nT - lT_s) + w(nT).
\]

\(^2\)The complexity of symbol timing synchronization can reduced somewhat in certain cases by sampling at 2 or 4 times the minimum sampling rate given by the sampling theorem.

\(^3\)Preference is given the the continuous-time definitions since the continuous-time signals are what is transmitted across the waveform channel.
The matched filter output is

\[ x_0(nT) = \sum_{m=n+\frac{T_2}{T}}^{n+T_2} \left( \sum_l a(l)p(mT - lT_s) \right) p(mT - nT) + \sum_{m=n+\frac{T_1}{T}}^{n+T_2} w(mT)p(mT - nT) \]

\[ = \sum_l a(l) \sum_{m=n+\frac{T_1}{T}}^{n+T_2} p(mT - lT_s)p(mT - nT) + \sum_{m=n+\frac{T_1}{T}}^{n+T_2} w(mT)p(mT - nT). \quad (2.42) \]

Using the substitution \( i = m - n \), the first summation in (2.42) can be expressed as

\[ \sum_{m=n+\frac{T_1}{T}}^{n+T_2} p(nT - lT_s)p(mT - nT) = \sum_{i=\frac{T_1}{T}}^{T_2} p(iT)p(iT - (lT_s - nT)) \]

\[ = \frac{1}{T} r_p(lT_s - nT) \quad (2.43) \]

where the last step follows from (2.37). Using \( v(nT) \) to represent the second term in (2.42), the matched filter output may be expressed as

\[ x_0(nT) = \frac{1}{T} \sum_l a(l)r_p(lT_s - nT) + v(nT). \quad (2.45) \]

The signal space projection is produced by taking every \( k\frac{T_2}{T} \)-th sample of the matched filter output. Evaluating (2.45) at \( n = k\frac{T_2}{T} \) gives

\[ x_0(kT_s) = \frac{1}{T} \sum_l a(l)r_p((l - k)T_s) + v(kT_s). \quad (2.46) \]

When the pulse shape satisfies the Nyquist no-ISI condition, the signal space projection is

\[ x(kT_s) = \frac{a(k)}{T} + v(kT_s). \quad (2.47) \]

The additive noise term moves the signal space projection \( x(kT_s) \) away from the true value \( a(k)/T \) as was the case with continuous-time processing. A decision error occurs when the noise moves \( x(kT_s) \) closer to a signal space point other than the one that was sent. The probability that this occurs is derived in Section 2.2.3.
Figure 2.9: A PAM modulator using discrete-time processing and a DAC (top) and a PAM matched filter detector using discrete-time processing with an ADC (bottom).
2.2.3 Performance

Bandwidth

For independent and equally likely symbols, the power spectral density of the PAM pulse train (2.21) is given by

\[ P_s(f) = \frac{\sigma_a^2}{T_s} |P(f)|^2 \]  

(2.48)

where \( P(f) \) is the continuous-time Fourier transform in \( f = \omega/2\pi \) of the pulse shape \( p(t) \). The power spectral density is determined by the properties of the Fourier transform of the pulse shape.

The bandwidth efficiency is a measure of how well a modulation uses the spectrum it occupies. The bandwidth efficiency is the ratio of the bit rate to the occupied bandwidth. Thus the bandwidth efficiency has units bits/second/Hz. The bandwidth efficiency is strongly dependent on the definition of bandwidth. For example, if the 90% bandwidth \( B_{90} \) is the measure of bandwidth, then the bandwidth efficiency for the NRZ, MAN, HS, and SRRC pulse shapes are

If the 99% bandwidth \( B_{99} \) is the measure of bandwidth, then the bandwidth efficiencies for the same pulse shapes is drastically reduced:

Probability of Error

The decision rule (2.14) partitions the signal space into decision regions as illustrated for the case of 4-ary baseband PAM in Figure 2.10. The probability of error is derived by first computing the conditional probabilities of error and applying the total probability theorem to give the desired result. In mathematical terms let \( s_0, s_1, \ldots, s_{M-1} \) be the \( M \) constellation points and let \( P(E|a = s_m) \) be the probability of error given the waveform corresponding to symbol \( s_m \) was sent for \( m = 0, 1, \ldots, M - 1 \). Applying the total probability theorem, the probability of error is

\[ P(E) = \sum_{m=0}^{M-1} P(E|a = s_m)P(a = s_m) \]  

(2.49)

where \( P(a = s_m) \) is the probability that symbol \( s_m \) was transmitted in the first place. Usually, it is assumed that the transmitted symbols are equally likely (i.e., neither the data nor the transmitter prefers sending one symbol over another). In this case

\[ P(a = s_m) = \frac{1}{M} \quad m = 0, 1, \ldots, M - 1 \]  

(2.50)

and (2.49) reduces to

\[ P(E) = \frac{1}{M} \sum_{m=0}^{M-1} P(E|a = s_m) \]  

(2.51)
All that remains is to compute the conditional probabilities of error.

The procedure for computing the conditional probabilities of error for $M$-ary PAM is demonstrated for the $M = 4$ case illustrated in Figure 2.10. The starting point is the signal space projection (2.45). Assuming $a(k) = +A$, $x(kT_s)$ is a random variable whose probability density function is illustrated in Figure 2.11 (a). The conditional probability of error $P(E|a(k) = +A)$ is the probability that $x(kT_s)$ is outside the decision region corresponding to $a(k) = +A$. As it turns out, the easiest way to compute this probability is one minus the probability that $x(kT_s)$ is inside the decision region corresponding to $a(k) = +A$. Following this line of reasoning,

$$P(E|a(k) = +A) = 1 - P \left( 0 \leq x(kT_s) \leq \frac{2A}{T} \right)$$

$$= 1 - P \left( 0 \leq \frac{A}{T} + v(kT_s) \leq \frac{2A}{T} \right)$$

$$= 1 - P \left( -\frac{A}{T} \leq v(kT_s) \leq \frac{A}{T} \right) \quad (2.52)$$

Note that the decision region boundaries have been scaled by $T$ since the signal space projection is also scaled by $T$. Applying the same procedure for the case $a(k) = -A$ produces

$$P(E|a(k) = -A) = 1 - P \left( -\frac{A}{T} \leq v(kT_s) \leq \frac{A}{T} \right) \quad (2.53)$$

which is the same as $P(E|a(k) = +A)$ since the decision regions are the same shape. Assuming $a(k) = +3A$, $x(kT_s)$ is a random variable whose probability density function is illustrated in Figure 2.11 (b). Following the same procedure as before

$$P(E|a(k) = +3A) = 1 - P \left( \frac{2A}{T} \leq x(kT_s) \right)$$

$$= 1 - P \left( \frac{2A}{T} \leq \frac{3A}{T} + v(kT_s) \right)$$

$$= 1 - P \left( -\frac{A}{T} \leq v(kT_s) \right) \quad (2.54)$$

$P(E|a(k) = +3A)$ is different from $P(E|a(k) = +A)$ since the decision regions have different shapes. The decision region for $a(k) = +3A$ is “open ended” while the decision region for $a(k) = +A$ is not. Following the same procedure once more, it can be shown that

$$P(E|a(k) = -3A) = 1 - P \left( -\frac{A}{T} \leq v(kT_s) \right) \quad (2.55)$$

which is the same as $P(E|a(k) = +3A)$ since the decision regions are the same shape.
The forgoing shows that the conditional probabilities of error reduce a computation involving the random variable \( v(kT_s) \) and the constant \( A \). The constant \( A \) is related to the received symbol energy. For 4-ary PAM with the four constellation points shown in Figure 2.11, the average symbol energy

\[
E_{\text{avg}} = 5A^2. 
\] (2.56)

Once the probability density function of \( v(kT_s) \) is known, evaluation of (2.52) – (2.55) is straightforward. The probability of density function of \( v(kT_s) \) is derived next.

It was shown in Chapter ?? that when \( w(t) \) is white Gaussian random process with zero mean and power spectral density \( N_0/2 \) Watts/Hz, \( T \)-spaced samples of the ideally bandlimited \( w(t) \) are zero mean Gaussian random variables with autocorrelation function

\[
E\{w(mT)w(nT)\} = \sigma^2\delta(m - n) 
\] (2.57)

where the variance is

\[
\sigma^2 = \frac{N_0}{2T}. 
\] (2.58)

The noise component at the matched filter output, \( v(nT) \) is given by

\[
v(nT) = \sum_{m=n+\frac{T_1}{T}}^{n+\frac{T_2}{T}} w(mT)p(mT - nT). 
\] (2.59)

Since \( v(nT) \) is a linear combination of the Gaussian random variables \( w(mT) \), \( v(nT) \) is also a Gaussian random variable. The joint probability density function is determined completely from the mean and auto-covariance of the sequence. The mean of each \( v(nT) \) is

\[
E\{v(nT)\} = E \left\{ \sum_{m=n+\frac{T_1}{T}}^{n+\frac{T_2}{T}} w(mT)p(mT - nT) \right\} = \sum_{m=n+\frac{T_1}{T}}^{n+\frac{T_2}{T}} E\{w(mT)\} p(mT - nT) = 0 
\] (2.60)
and the auto-covariance is given by

\[
E\{v(nT)v(lT)\} = E \left\{ \sum_{m=n+n_T}^{n+n_T} w(mT)p(mT - nT) \sum_{m'=l+l_T}^{l+l_T} w(m'T)p(m'T - lT) \right\}
\]

\[
= \sum_{m=n+n_T}^{n+n_T} \sum_{m'=l+l_T}^{l+l_T} E\{w(mT)w(m'T)\} p(mT - nT)p(m'T - lT)
\]

\[
= \sigma^2 \sum_{m=n+n_T}^{n+n_T} p(mT - nT)p(mT - lT)
\]

\[
= \sigma^2 r_p(-(n - l)T). \quad (2.61)
\]

The noise component in the signal space projection is a downsampled version of \(v(nT)\). Clearly \(v(kT_s)\) has zero mean. The correlation is

\[
E\{v(kT_s)v(lT_s)\} = \frac{\sigma^2}{T} r_p(-(k - l)T_s)
\]

\[
= \frac{\sigma^2}{T} \delta(k - l). \quad (2.62)
\]

where the last line is true only if the pulse shape satisfies the Nyquist no-ISI condition. In summary, the random variables \(v(kT_s)\) are independent and identically distributed random variables with zero mean and variance

\[
\sigma^2 \nu = \frac{\sigma^2}{T} = \frac{N_0}{2T^2}. \quad (2.63)
\]

Now back to the the probability of error for 4-ary PAM. The probabilities (2.52) – (2.55) can be expressed in terms of the area under the upper tail of the standard Gaussian random variable, denoted by the \(Q\)-function as described in Chapter ??:

\[
P(E|a(k) = +A) = P\left( E|a(k) = -\frac{A}{T} \right) = 2Q\left( \frac{A}{\sigma_v} \right) = 2Q\left( \sqrt{\frac{2A^2}{N_0}} \right) \quad (2.65)
\]

\[
P(E|a(k) = +3A) = P\left( E|a(k) = -\frac{3A}{T} \right) = Q\left( \frac{A}{\sigma_v} \right) = Q\left( \sqrt{\frac{2A^2}{N_0}} \right). \quad (2.66)
\]
Note that the last equality follows from the relationship (2.64) Applying the total probability theorem assuming equally likely symbols gives

$$P(E) = \frac{3}{2} Q\left(\sqrt{\frac{2 A^2}{N_0}}\right).$$  \hspace{1cm} (2.67)

Now using (2.56), the probability of error may be expressed as

$$P(E) = \frac{3}{2} Q\left(\sqrt{\frac{2 E_{\text{avg}}}{5 N_0}}\right).$$  \hspace{1cm} (2.68)

This shows that when a matched filter detector is used with perfect timing synchronization, the probability of symbol error is a function only of the ratio of the average symbol energy and the power spectral density level of the white noise.

Two normalizations are commonly used to facilitate comparisons between modulations with different symbol sizes. The first is to reduce the symbol error probability to a bit error probability. This is due to the notion that the fundamental quantity being transmitted is a bit, no matter how the bits are mapped into symbols. As a consequence, the fundamental measure of reliability is the bit error rate\(^4\). The translation from symbol error probability to bit error probability follows from the observation that the most likely symbol errors are those involving adjacent symbols. Using a gray code, the bit patterns associated with adjacent symbols differ in only one position. Hence, the most likely symbol errors produce one erroneous bit and \(\log_2(M) - 1\) correct bits. Thus

$$P_b = \frac{1}{\log_2 M} P(E).$$ \hspace{1cm} (2.69)

The translation from symbol error probability to bit error probability can be thought of as a scaling of the “y-axis” in a probability of error versus signal-to-noise ratio plot. The scaling of the “x-axis” in such a plot involves scaling the signal-to-noise ratio. Since the bit is the fundamental quantity being transmitted, the bit energy is the fundamental unit of energy. The average bit energy, \(E_b\), is related to the average symbol energy, \(E_{\text{avg}}\), by

$$E_{\text{avg}} = (\log_2 M) \times E_b.$$  \hspace{1cm} (2.70)

Applying these two normalizations to (2.68) produces

$$P_b = \frac{3}{4} Q\left(\sqrt{\frac{4 E_b}{5 N_0}}\right).$$ \hspace{1cm} (2.71)

\(^4\)For a communication system where the data are grouped into frames or packets, many prefer the frame error rate or packet error rate as the fundamental measure of reliability. This follows from the notion that if one or more bits in a packet is in error, then none of the bits in the packet is of use to the end user.
Generalizing the forgoing to $M$-ary PAM, the probability of symbol error and bit error are

$$P(E) = 2 \frac{M-1}{M} Q \left( \sqrt{\frac{6}{M^2 - 1}} \frac{E_{av}}{N_0} \right)$$  \hspace{1cm} (2.72)

$$P_b = \frac{2(M-1)}{M \log_2 M} Q \left( \sqrt{\frac{\log_2 M E_b}{M^2 - 1} N_0} \right).$$  \hspace{1cm} (2.73)

The probability of bit error (2.73) for $M = 2, 4, \text{ and } 8$ is plotted in Figure 2.12. A vertical slice through the plot is the line of constant bit energy. The probability of bit error increases as $M$ increases since the average symbol energy must be shared with more bits. A horizontal line through the plot is the line of constant reliability. For a fixed probability of bit error, more bit energy is needed as $M$ increases. For example, at $P_b = 10^{-6}$ binary PAM requires $E_b/N_0 = 10.6 \text{ dB}$, 4-ary PAM requires $E_b/N_0 = 14.6 \text{ dB}$, and 8-ary PAM requires $E_b/N_0 = 19.2 \text{ dB}$. 4-ary PAM is said to be 4 dB worse than binary PAM at $P_b = 10^{-6}$ and 8-ary PAM is 8.6 dB worse than binary PAM at $P_b = 10^{-6}$.

However, these conclusions ignore bandwidth. 4-ary PAM is 2 times more bandwidth efficient than binary PAM and 8-ary PAM is 3 times more bandwidth efficient than binary PAM. This can be seen in one of two ways: assuming the bit rate is constant so that the bandwidth varies with $M$, or by assuming the bandwidth is constant so that the bit rate varies with $M$. Either assumption leads to the same conclusion. The bandwidth efficiency is measured in bits/second/Hz as described above.
2.2 M-ary Baseband Pulse Amplitude Modulation (PAM)

Figure 2.10: The four decision regions for 4-ary PAM.

Figure 2.11: Probability density function for matched filter output (a) when $a/T = +A$ and (b) when $a/T = +3A$. 
Figure 2.12: Probability of bit error versus $E_b/N_0$ for M-ary PAM for $M = 2$, 4, and 8.